Last day

If we have an inhomogeneous system with solution

$$\vec{x} = \vec{q} + \alpha_1 \vec{a_1} + \alpha_2 \vec{a_2} + \cdots + \alpha_n \vec{a_n}$$

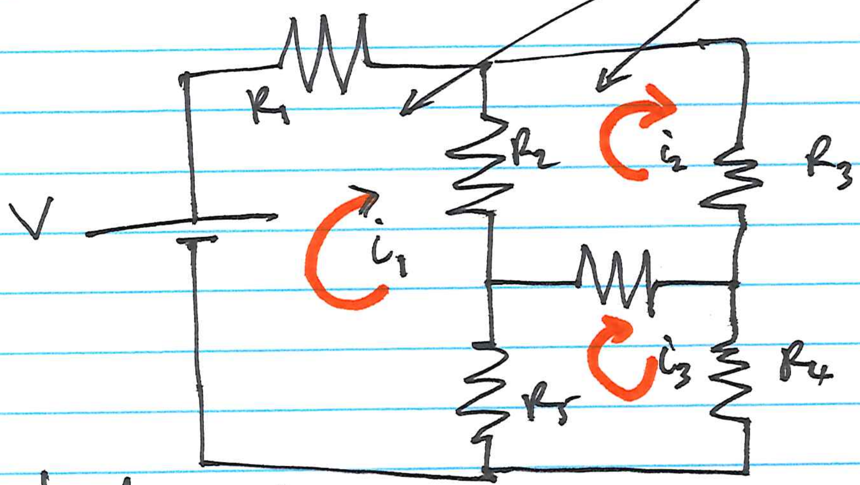
$\alpha_1, \alpha_2, \ldots, \alpha_n$ are scalars.

If $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$, then $\vec{x} = \vec{q}$ is also a solution of the system.

Loop currents

— ~~elimin~~

elementary closed loops.

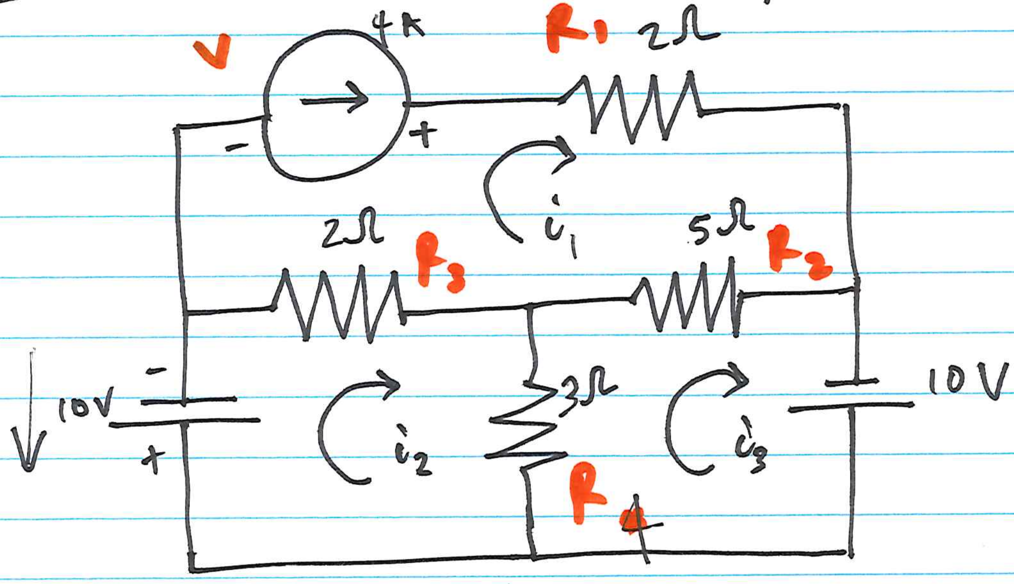


This technique:

— eliminates redunancy

— give unique solution to the system of equations.

Example! Solve the following ~~thek~~ network



find the current through ~~each~~ each elementary loop and the voltage through each current source.

- our unknown variables are
  v, i_1, i_2, i_3

From loop 1!

$V = IR$

$V = 2i_1 + 5(i_1 - i_3) + 2(i_1 - i_2)$

from R_1 : $2i_1$

from R_2 : $5(i_1 - i_3)$

$9i_1 - 2i_2 - 5i_3 - V = 0$ — (1)

from R_3 : $2(i_1 - i_2)$

From loop 2:

$$-10 = 2(i_2 - i_1) + 3(i_2 - i_3)$$

$$= 2i_2 - 2i_1 + 3i_2 - 3i_3$$

$$-10 = -2i_1 + 5i_2 - 3i_3$$

$$-2i_1 + 5i_2 - 3i_3 = -10 \quad \text{——— } (2)$$

From loop 3:

$$10 = 3(i_3 - i_2) + 5(i_3 - i_1)$$

$$-5i_1 - 3i_2 + 8i_3 = 10 \quad \text{——— } (3)$$

- look at the ~~power~~ current source.

$$i_1 = 4$$

$$9i_1 - 2i_2 - 5i_3 - V = 0$$

$$-2i_1 + 5i_2 - 3i_3 = -10$$

$$-5i_1 - 3i_2 + 8i_3 = 10$$

$$i_1 = 4$$

the augmented matrix is

$$A = \left(\begin{array}{cccc|c} 9 & -2 & -5 & -1 & 0 \\ -2 & 5 & -3 & 0 & -10 \\ -5 & -3 & 8 & 0 & 10 \\ 1 & 0 & 0 & 0 & 4 \end{array}\right)$$

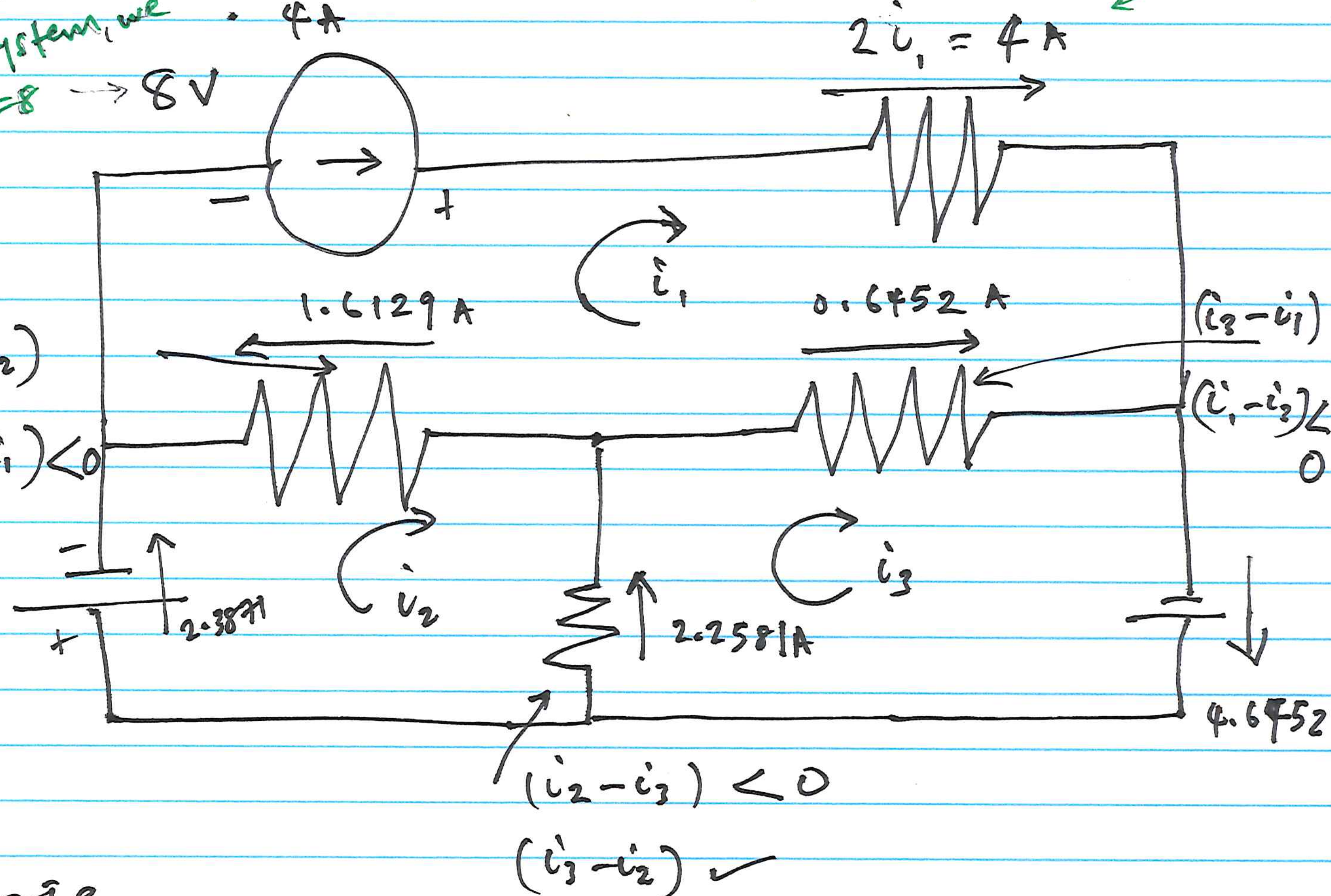
using the command rref(A) in MATLAB, it gives

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 2.3871 \\ 0 & 0 & 1 & 0 & 4.6452 \\ 0 & 0 & 0 & 1 & 8 \end{array}\right)$$

$$\Rightarrow \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ v \end{pmatrix} = \begin{pmatrix} 4 \\ 2.3871 \\ 4.6452 \\ 8 \end{pmatrix}$$

$V = IR$
$V = 8,\ R = 2$ $\Rightarrow i = 8/2 = 4A$

from the solution of the system, we have $v = 8 \rightarrow 8V$

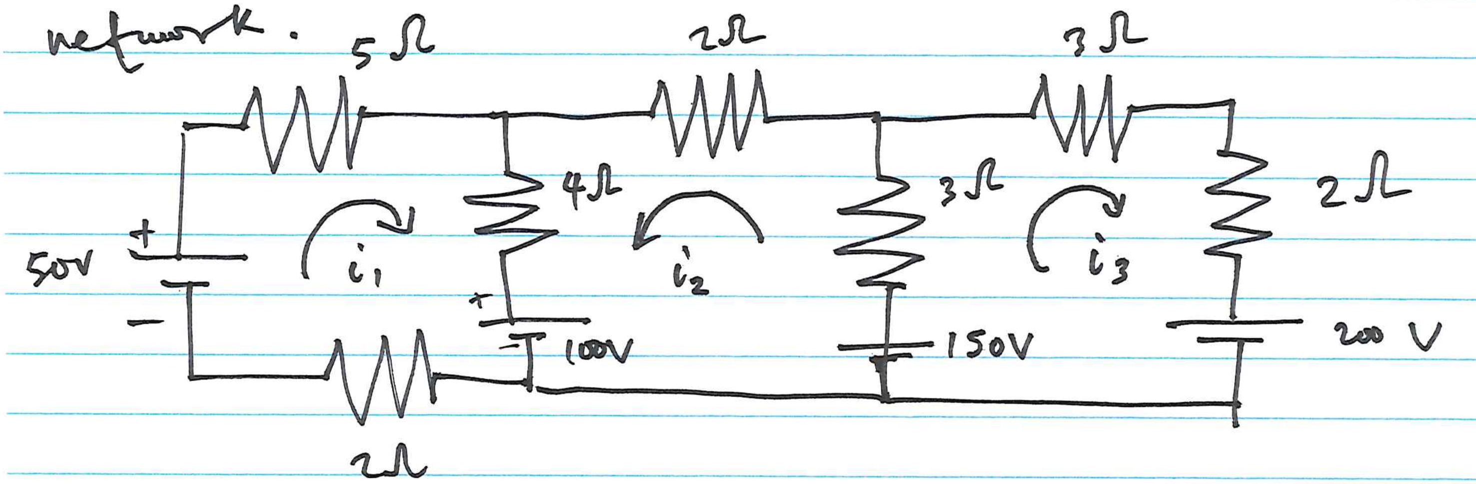


$V = \cancel{IR}$

$I = V/R$

Example:

Find the current in each of the branches of the network.



our ~~unknowns~~ unknowns are i_1, i_2, and i_3

From loop 1:

$$-100 + 50 = 5i_1 + 4(i_1 + i_2) + 2i_1$$

Simplifying,

$$11 i_1 + 4 i_2 = -50 \quad (1)$$

From loop 2:

$$150 - 100 = 3(i_2 + i_3) + 2i_2 + 4(i_2 + i_1)$$

$$\Rightarrow \quad 4i_1 + 9i_2 + 3i_3 = 50 \quad (2)$$

From loop 3:

$$150 - 200 = 3(i_2 + i_3) + 3i_3 + 2i_3$$

$$3i_2 + 8i_3 = -50 \qquad \text{③}$$

$$11i_1 + 4i_2 = -50$$

$$4i_1 + 9i_2 + 3i_3 = 50$$

$$3i_2 + 8i_3 = -50$$

the augmented matrix is

$$\left(\begin{array}{ccc|c} 11 & 4 & 0 & -50 \\ 4 & 9 & 3 & 50 \\ 0 & 3 & 8 & -50 \end{array}\right)$$

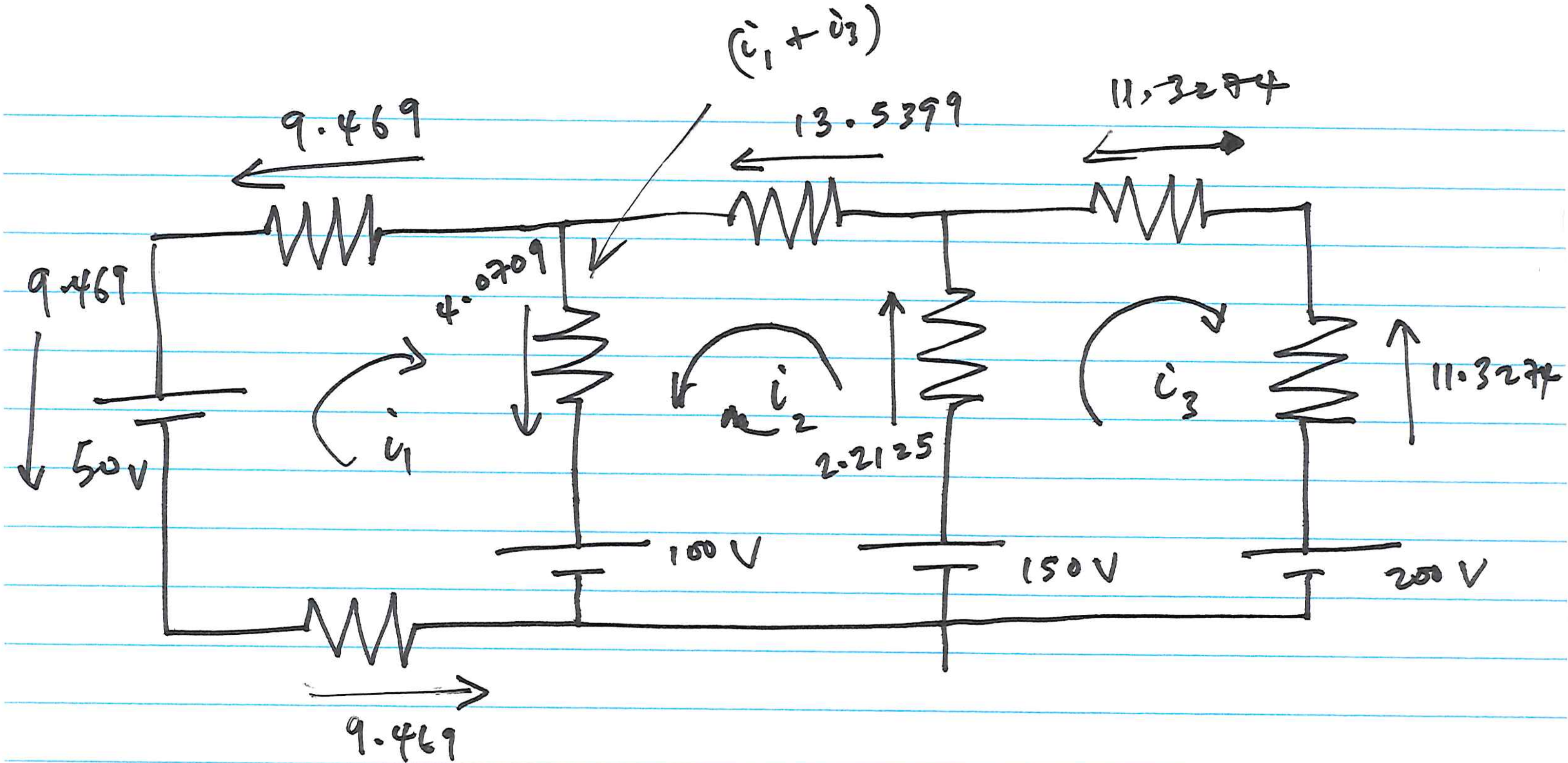
using rref in MATLAB, we have

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -9.469 \\ 0 & 1 & 0 & 13.5399 \\ 0 & 0 & 1 & -11.3274 \end{array}\right)$$

$$\Rightarrow \quad i_1 = -9.469$$

$$i_2 = 13.5399$$

$$i_3 = -11.3274$$


(i1 + i3)
9.469
13.5399
11.3274
9.469
4.0709
i1
i2
i3
2.2125
11.3274
50V
100 V
150 V
200 V
9.469

~~V = IR~~

~~I = V/R~~

$a_{1,1}$

MATRIX OPERATIONS

Let A be an $n \times m$ matrix

number of rows — number of columns

E.g

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{pmatrix}$$

<u>~~Column~~ Row vector</u>: an $1 \times n$ matrix is a ~~column~~ row vector

E.g $(a_1, a_2, \ldots, a_n)$

<u>Column vector</u>: an $m \times 1$ matrix is a column vector

E.g $\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix}$

- <u>Matrix addition:</u>

Let $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$

$B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}$

$A+B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{pmatrix}$

- Scalar multiplication

Let α be scalar and A be the same as the previous matrix.

Then

$$\alpha A = \alpha \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha a_{11} & \alpha a_{12} & \cdots & \alpha a_{1n} \\ \alpha a_{21} & \alpha a_{22} & \cdots & \alpha a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha a_{m1} & \alpha a_{m2} & \cdots & \alpha a_{mn} \end{pmatrix}$$