MATH 152 – Linear Systems
Test #1, Version B (TTh 8am sections)

Spring, 2017
University Of British Columbia

Name: ____

ID Number:____

Section: ____

Instructions

- **You should have seven pages including this cover.**
- **There are 2 parts to the test:**
  - **Part A has 10 short questions worth 1 mark each**
  - **Part B has 3 long questions worth 5 marks each**
- **Although all questions in each part are worth the same, some may be more difficult than others – do the easy questions first!**
- **Use this booklet to answer questions.**
- **Return this exam with your answers.**
- **Please show your work. Correct intermediate steps may earn credit.**
- **No calculators are permitted on the test.**
- **No notes are permitted on the test.**
- **Maximum score = 25 Marks (attempt all questions)**
- **Maximum Time = 50 minutes.**

GOOD LUCK!

Part A Total	B1	B2	B3	Total
10	5	5	5	25

Part A - Short Answer Questions, 1 mark each

For Questions A1-A3 below, let

$$\begin{aligned}\mathbf{a} &= [1, 1, 5] \\ \mathbf{b} &= [2, -2, 1]\end{aligned}$$

The answers for questions A1-A2 should be in the form $[x, y, z]$ with x, y, and z determined.

A1: Compute $\mathbf{a} \times \mathbf{b}$

A2: Compute $\text{proj}_{\mathbf{a}}\mathbf{b}$

A3: Determine the value of $\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. *Note:* do not try to determine the value of θ.

A4: Give two vectors of length 10 that are parallel to $[3, 1, 5]$.

A5: Consider the following lines of MATLAB code:

```
A = zeros(3,2);
for i=1:2
   A(i,i) = i
end
```

What is the result in `A`?

A6: Consider a linear system of eight equations in six unknowns. Circle below *all possible* forms of the solution set to the system.

(a) The system might have no solutions

(b) The system might have exactly one solution

(c) The system might have exactly two solutions

(d) The system might have an infinite number of solutions

(e) The set of solutions might be three dimensional

A7: Determine whether the lines L_1 and L_2 below intersect. If they do, find the point of intersection. If they do not, justify your answer.

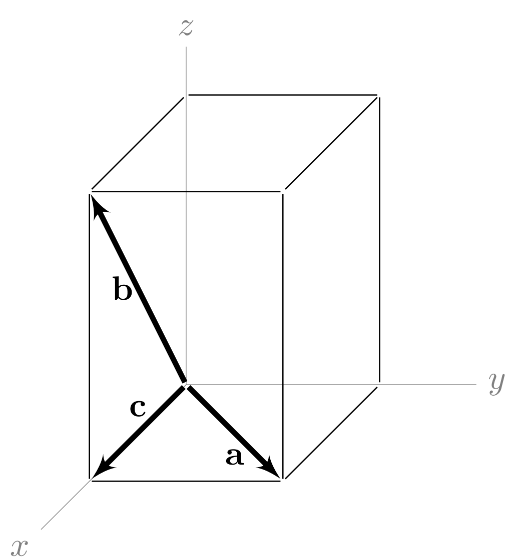
$$\begin{aligned} L_1 &: \mathbf{x} = [3,0,2] + s[-1,1,2] \\ L_2 &: \mathbf{x} = [0,0,7] + t[1,2,-1] \end{aligned}$$

A8: Find the distance from the origin to the plane $3x - y - 4z = -2$.

A9: Below is an augmented matrix representing a system of linear equations. Describe all solutions to the system of equations in parametric form.

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 4 & 3 & -1 & 3 \\ 0 & 1 & 0 & 2 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

A10: Below is a picture of a rectangular box in $\mathbb{R}^3$. The farthest corner from the origin is (p_1, p_2, p_3). Write $(0, 0, p_3)$ as a linear combination of the named vectors in the picture.



Part B - Long Answer Questions, 5 marks each

B1: Consider the linear system below for the unknowns x, y and z. It is known that the system has a unique solution.

$$\begin{aligned} 2x - 2y + 6z &= 4 \\ -2x + 3y - z &= 5 \\ 6x - 5y + 3z &= 1 \end{aligned}$$

(a) [1 mark] Write the system in an augmented matrix.

(b) [3] Do row operations (Gaussian elimination) on the augmented matrix to change it to echelon form.

(c) [1] Find the solution to the problem from the form above.

B2: For each of the systems described below as augmented matrices that have been put into reduced row echelon form, determine whether the systems have

- no solutions
- a single solution (in this case, find the solution)
- an infinite number of solutions (in this case, find a parametric description of all solutions).

Each part is worth 1 mark.

(a) $\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 0 & 3 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 7 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 5 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$

(d) $\left[\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(e) $\left[\begin{array}{cccc|c} 1 & 5 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 10 \end{array}\right]$

B3: Given the vectors

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 6 \\ 2 \\ 10 \end{bmatrix}.$$

(a) [1 mark] Show that $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are linearly independent.

(b) [2] Find all possible combinations of scalars $\{x_1, \ x_2, \ x_3\}$ such that

$$x_1\mathbf{a} + x_2\mathbf{b} + x_3\mathbf{c} = \mathbf{v}.$$

(c) [2] Let S_1 be the plane that contains vectors $\mathbf{a}$ and $\mathbf{b}$ and the origin; and S_2 be the plane that contains vectors $\mathbf{c}$ and $\mathbf{v}$ and the origin. Find the parametric equation of the line of intersection between S_1 and S_2.