MATH 152 – Linear Systems
Test #2, Version A (TTh sections)

Spring, 2017
University Of British Columbia

Family Name: ___________________ **Given Name:** ___________________

Student ID: ______________________ **Section:** ________________________

Instructions

- **You should have seven pages including this cover.**
- **There are 2 parts to the test:**
  - **Part A has 10 short questions worth 1 mark each**
  - **Part B has 3 long questions worth 5 marks each**
- **Although all questions in each part are worth the same, some may be more difficult than others – do the easy questions first!**
- **Use this booklet to answer questions.**
- **Return this exam with your answers.**
- **Please show your work. Correct intermediate steps may earn credit.**
- **No calculators are permitted on the test.**
- **No notes are permitted on the test.**
- **Maximum score = 25 Marks (attempt all questions)**
- **Maximum Time = 50 minutes.**

GOOD LUCK!

Part A Total	B1	B2	B3	Total
10	5	5	5	25

Part A - Short Answer Questions, 1 mark each

A1: Compute the determinant of the matrix

$$\begin{bmatrix} 10 & 19 & 8 & 53 \\ 0 & 1 & 19 & 2 \\ 0 & 0 & \frac{1}{5} & 4 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

A2: Given that $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \end{bmatrix}$, and $\mathbf{x} = [2\ 1]$, calculate $(\mathbf{x}A)^T$.

A3: For $A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$, find A^{-1}.

A4: Which of the following are true for *all* 4×4 matrices A, with $\det(A) = 10$? Circle all that apply.

(a) The reduced row echelon form of A is the 4×4 identity matrix.

(b) A is invertible.

(c) The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

(d) The rank of A is 10.

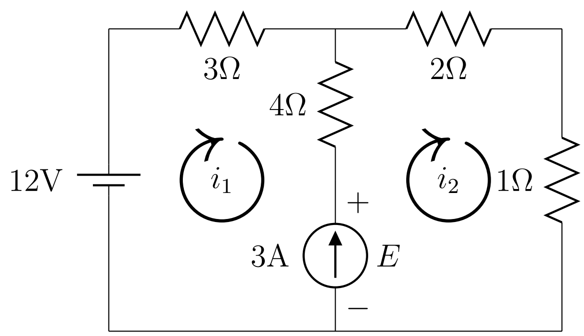
(e) $A = A^T$.

A5: Consider the following lines of MATLAB code:

```
A = [1 1; 1 1];
rref(A)
```

What is the result of the last line above?

Questions A6-A7 concern the resistor network below.



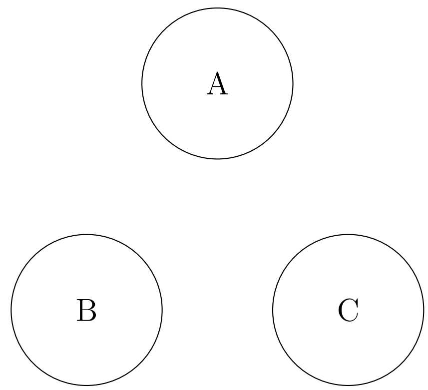
A6: Write the linear equation involving the unknown loop currents and current source voltage that corresponds to summing the voltage drops around loop 1 (Kirchhoff's second law).

A7: Write the linear equation that matches the loop currents to the current source.

A8: Suppose the matrix A below satisfies $A^T = A^{-1}$. What are all possible values of c and d?

$$A = \begin{bmatrix} 0 & 1 \\ c & d \end{bmatrix}$$

Questions A9-A10 below concern a game described as follows: You place a game piece on one of the three circles below. You roll a 6-sided die to figure out where your game piece moves. If you roll 1, you stay in your space. If you roll 2 or 3, you move your piece clockwise. If you roll 4, 5, or 6, you move your piece counterclockwise.



A9: Write a probability transition matrix P for the game. Use the ordering ABC for the states.

A10: Suppose you start on Circle A (before you've rolled the die at all). What is the probability that after two rolls, your piece is in Circle A?

Part B - Long Answer Questions, 5 marks each

B1: Match the matrices A below with their inverse. One mark each.

(a) $$A = \begin{bmatrix} -2 & -3 & -1 \\ -1 & -2 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

(b) $$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(c) $$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

(d) $$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \end{bmatrix}$$

(e) $$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

(A) A^{-1} does not exist.

(B) $$A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & -1 \\ 3 & -5 & 1 \end{bmatrix}$$

(C) $$A^{-1} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

(D) $$A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \end{bmatrix}$$

(E) $$A^{-1} = \begin{bmatrix} 4/3 & -1/2 \\ 1/3 & 0 \\ -2/3 & 1/2 \end{bmatrix}$$

(F) A^{-1} is not in the list above.

B2: Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. Given that

$$T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

- **(a)** [1 mark] Write the vector [1 0 0] as a linear combination of [1 1 1] and [0 1 1].
- **(b)** [3] Let B be the matrix representation of T. Find B.
- **(c)** [1] Determine whether B is invertible using a determinant calculation.

B3: Let $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that reflects a vector across the line $y = x$, and let $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that rotates a vector $45°$ counter-clockwise.

(a) [1 mark] Write a matrix R_1 such that $T_1(\mathbf{x}) = R_1\mathbf{x}$ for any $\mathbf{x} \in \mathbb{R}^2$.

(b) [1] Write a matrix R_2 such that $T_2(\mathbf{x}) = R_2\mathbf{x}$ for any $\mathbf{x} \in \mathbb{R}^2$.

(c) [1] Suppose $T_3 = T_1 \circ T_2$. That is, T_3 is the transformation that takes an input vector in $\mathbb{R}^2$, rotates it counter-clockwise, then reflects it. Find a matrix R_3 such that $T_3(\mathbf{x}) = R_3\mathbf{x}$ for any $\mathbf{x} \in \mathbb{R}^2$.

(d) [2] Find a nonzero vector $\mathbf{x}$ in $\mathbb{R}^2$ such that $T_1(\mathbf{x}) = T_2(\mathbf{x})$, or show with a calculation that none exists.