LIST OF TOPICS: MATH 215/255 SEPTEMBER-DECEMBER 2017

Important Notes: This list of topics may be slightly modified as the term progresses. You will be notified and a specific list of topics will be provided before each test and before the final exam. References to sections of Lebl are provided for convenience but the material in Lebl will often need to be supplemented by reference to course notes, homework problems, homework solutions and other materials in order to achieve full understanding. Also, we will not necessarily cover the course topics in the order they appear here.

First Order Differential Equations

- Identify linear, nonlinear and separable equations, and understand order of an ODE (Lebl 0.2, 0.3, 1.3).

- Understand solutions in algebraic, integral and graphical forms (Lebl 1.1).

- Know and apply conditions that guarantee the existence and uniqueness of solutions for first order equations (Lebl 1.2).

- Understand slope fields for first order equations and be able to plot and interpret simple examples (Lebl 1.2).

- Understand and interpret plots of dy/dx vs y for autonomous equations (Lebl 1.6).

- Solve 1st order linear differential equations using an integrating factor. (Lebl 1.4)

- Solve separable 1st order equations via separation of variables (Lebl 1.3).

- Interpret solutions and link to properties of the original ODE (Examples in class and assignments).

- Applications, including: Newton's law of cooling, dilutions, evaporation problems, population growth, logistic equation, radioactivity, etc (Examples in class, assignments).

- Identify and solve exact equations (Lebl 1.8).

Numerical Methods

- Know Euler's method and apply to find numerical approximations to solutions of differential equations. (Lebl 1.7)

- Understand the estimated convergence of Euler's method (global (accumulated) error proportional to the time step h, for small h). (Lebl 1.7)

- Understand Matlab/Octave syntax as used on the homework problems (plotting direction fields for 1-D problems, vector fields for 2-D problems, and using ode45 to generate numerical solutions) (assignments)

Systems of Linear First Order Differential Equations

- Solve simple systems using eigenvalue-eigenvector method including complex eigenvalue and repeated eigenvector (defective matrix) cases (Lebl 3.4, 3.7).

- Understand direction vector fields for first order equations and be able to plot and interpret for all 2x2 cases (Lebl 3.5).

- Understand and apply matrix exponential to solve systems where the matrix is diagonalizable (Lebl 3.8).

- Interpret solutions and link to properties of the original system, including applications (Examples in class and assignments).

Forced Systems of Linear Equations ($\vec{x}' = A\vec{x} + \vec{h}(t)$)

- Be able to solve simple 2x2 systems with forcing (Lebl 3.9), including selecting the best method to use.

- Interpret solutions and link to properties of the original system, including applications (Lebl 3.9., Examples in class).

Nonlinear Autonomous Systems of Differential Equations

- Identify nonlinear autonomous systems of differential equations (Lebl 8.1).

- Find critical points of such a system. Using the Jacobian matrix, describe the local behaviour of the solutions near critical points (Lebl 8.1).

- Interpret and plot qualitative solutions and link to properties of the original system (Lebl 8.2).

- Modelling applications: disease spread, competing species, predator-prey and nonlinear pendulum. *You will not need to set up the equations.* (Lebl 8.3, additional notes, homework)

Second-order constant-coefficient differential equations ($y'' + by' + cy = f(t)$)

- Identify and solve homogeneous and non-homogeneous (forced) second order equations by finding homogeneous and particular solutions (Lebl 2).

- Understand and apply the main results on existence and uniqueness of solutions for linear differential equations (Lebl 2.1, 2.3).

- Rewrite second order differential equations as 2x2 systems of first order equations (Class notes).

- Find particular solutions by Undetermined Coefficients method (aka guess-and-check) or by re-writing as a system of equations (Lebl 2.5, class notes).

- Interpret solutions for simple applications (e.g. spring-mass, LCR circuit, linear pendulum) in the presence or absence of damping and forcing terms. You will not have to derive the governing equations for electrical circuits or pendulums (Lebl 2, class notes).

- Identify and understand *resonance* and *beats* in sinusoidally-forced oscillatory systems with and without damping. Calculate and plot the amplitude of the steady solution (the *frequency response*) for such a system (Lebl 2.6).

- Make reasonably accurate plots of solutions (Lebl 2.4, 2.6 and class notes).

Laplace Transform for Differential Equations

- Define the Laplace transform and understand the basics of how to use it to solve differential equations (Lebl 6.1, 6.2).

- Calculate the Laplace transforms of simple functions by direct integration or by using a table of transforms. (Lebl 6.2, class notes)

- Solve simple Laplace transform problems using partial fractions, completing the square etc (Lebl 6.2, class notes).

- Solve ODEs with Laplace transform, including delta-function forcing (Lebl 6.4).

- Calculate and apply system transfer functions and impulse response (Lebl 6.4).

- Convolution integrals as inverse transforms (Lebl 6.4).

Note: a table of Laplace Transforms will be included on tests and exams as needed

Simpler Version
To a first approximation, we will cover the following sections:

- Introduction 0.2, 0.3

- Chapter 1: 1.1-1.8

- Chapter 2: 2.1-2.6

- Chapter 3: 3.1-3.9

- Chapter 6: 6.1-6.4

- Chapter 8: 8.1-8.3 (also see Stephen Gustafson's notes on nonlinear systems)