

LIST OF TOPICS: MATH 215/255 SEPTEMBER-DECEMBER 2017 MIDTERM TWO

Note: The second midterm is cumulative. Any topic from any part of the course so far could be tested either individually or in combination with other course topics. Make sure you can do all webwork, homework, quiz and midterm one problems.

First Order Differential Equations

- Identify linear, nonlinear and separable equations, and understand order of an ODE (Lebl 0.2, 0.3, 1.3).
- Understand solutions in algebraic, integral and graphical forms (Lebl 1.1).
- Know and apply conditions that guarantee the existence and uniqueness of solutions for first order equations (Lebl 1.2).
- Understand slope fields for first order equations and be able to plot and interpret simple examples (Lebl 1.2).
- Understand and interpret plots of dy/dx vs y for autonomous equations (Lebl 1.6).
- Solve 1st order linear differential equations using an integrating factor. (Lebl 1.4)
- Solve separable 1st order equations via separation of variables (Lebl 1.3).
- Interpret solutions and link to properties of the original ODE (Examples in class and assignments).
- Applications, including: Newton's law of cooling, dilutions, evaporation problems, population growth, logistic equation, radioactivity, etc (Examples in class, assignments).
- Identify and solve exact equations (Lebl 1.8).

Numerical Methods

- Know Euler's method and apply to find numerical approximations to solutions of differential equations. (Lebl 1.7)
- Understand the estimated convergence of Euler's method (global (accumulated) error proportional to the time step h , for small h). (Lebl 1.7)
- Understand Matlab/Octave syntax as used on the homework problems (plotting direction fields for 1-D problems, and using ode45 to generate numerical solutions) (assignments)

Systems of Linear First Order Differential Equations

- Solve simple systems using eigenvalue-eigenvector method including complex eigenvalue and repeated eigenvector (defective matrix) cases (Lebl 3.4, 3.7).
- Understand vector fields for first order equations and be able to plot and interpret for all 2x2 cases (Lebl 3.5).
- Understand and apply matrix exponential to solve systems where the matrix is diagonalizable (Lebl 3.8).

- Interpret solutions and link to properties of the original system, including applications (Examples in class and assignments).
- Compute and work with the fundamental matrix for constant-coefficient problems.

Forced Systems of Linear Equations ($\vec{x}' = A\vec{x} + \vec{h}(t)$)

- Be able to solve simple 2x2 systems with forcing (Lebl 3.9) using method of undetermined coefficients, fundamental matrix, and matrix exponential.

Second-order constant-coefficient differential equations ($y'' + by' + cy = f(t)$)

- Identify and solve homogeneous and non-homogeneous (forced) second order equations by finding homogeneous and particular solutions (Lebl 2).
- Understand and apply the main results on existence and uniqueness of solutions for linear differential equations (Lebl 2.1, 2.3).
- Rewrite second order differential equations as 2x2 systems of first order equations (Class notes).
- Find particular solutions by Undetermined Coefficients method (aka guess-and-check) or by re-writing as a system of equations and applying fundamental matrix method (Lebl 2.5, class notes).
- Interpret solutions for simple applications (e.g. spring-mass, LCR circuit, linear pendulum) in the presence or absence of damping and forcing terms. You will not have to derive the governing equations for electrical circuits or pendulums (Lebl 2, class notes).
- Identify and understand *resonance* and *beats* in sinusoidally-forced oscillatory systems with and without damping. Calculate and plot the amplitude of the steady solution (the *frequency response*) for such a system (Lebl 2.6).
- Make reasonably accurate plots of solutions (Lebl 2.4, 2.6 and class notes).