Slope fields

Given an ODE of the form

$$y' = f(t, y)$$

The ~~for~~ unction $f(t, y)$ ~~is~~ gives the slope of the solution of the ODE at each point (t, y)

Example: sketch the slope field of

$$dy/dt = (2-t)\,y \quad \text{on} \quad \begin{array}{l} -4 \le t \le 4 \\ -4 \le y \le 4 \end{array}$$

$f(t, y) = (2-t)y$

for $t = 0$,

$f(0, y) = 2y$,

for $t = 1$

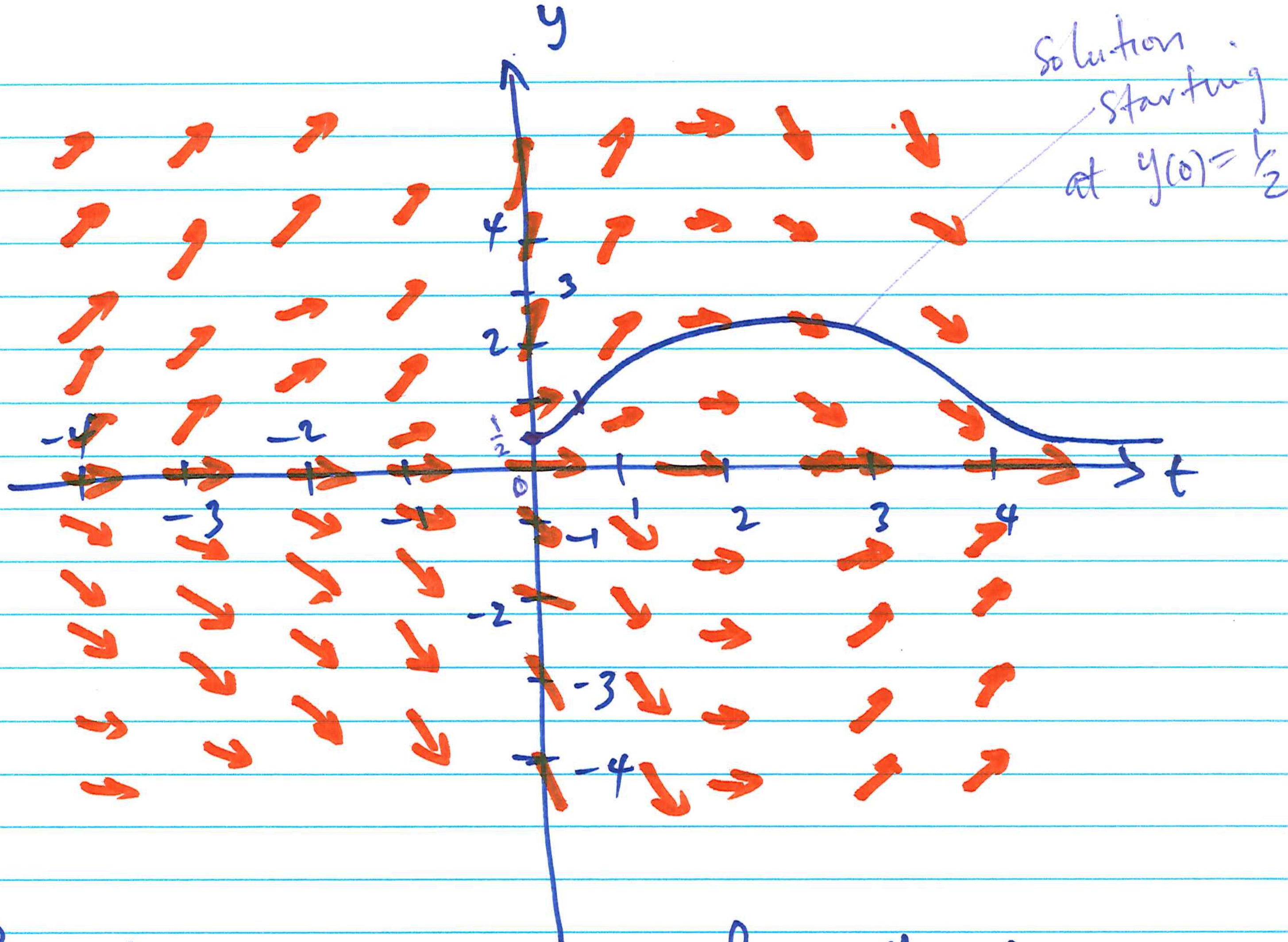
$f(1, y) = y$

for $y = 0$,

$f(t, 0) = 0$

for $t = 2$

$f(2, y) = 0$



for all $t > 2$

$f > 0$ if $y < 0$

$f < 0$ if $y > 0$

for all $t < 0$

$f > 0$ if $y > 0$

$f < 0$ if $y < 0$

* Draw the solution of the equation that has the initial condition

$$y(0) = \frac{1}{2}$$

~~Au~~ Autonomous equations - are ~~d~~ equations of the form

$$y' = f(y)$$

where the function f is ~~independen~~ dependent only on the dependent variable. That is, f is function of y only.

Example: (i) $$\frac{dy}{dt} = 4y$$

$$f = f(y)$$

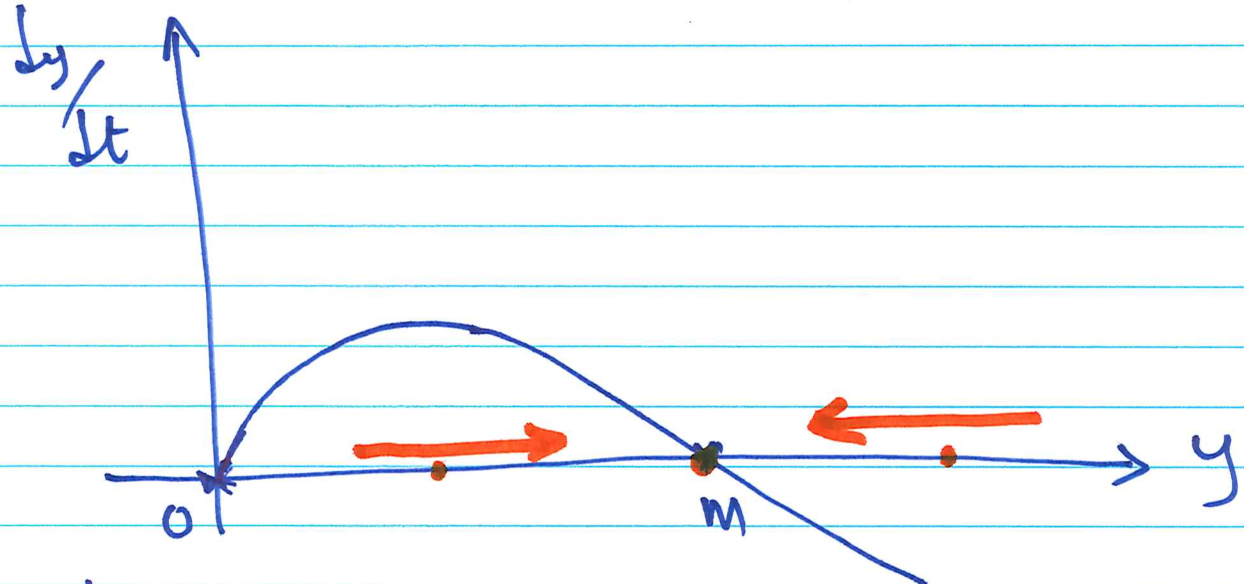
(ii) $$\frac{dy}{dt} = ky(m-y)$$

Logistic equation used in population modelling.

k - population growth rate

M - maximum sustainable population.

Let us plot dy/dt vs y



$dy/dt = 0 \rightarrow \underset{ss}{y=0}$, $\underset{ss}{y=M}$

$y_{ss} = 0$ and $y_{ss} = M$

are called <u>steady-state</u> solutions of the ODE (equilibrium solution / fixed point).

If we solve the ODE with an I.C $y_0 < M$ the solution converges to ~~M~~ $y=M$.

also starting with $y_0 > M$ the solution converges to $y = M$.

If we start with $y_0 = 0$ or $y_0 = M$,
the solutions will remain there because
they are steady-state solutions of
the system.

~~Also the solutions conve~~

Since the solution converge to $y_{ss} = M$,
~~as~~ we say $y_{ss} = M$, is stable.
and since it diverges from $y_{ss} = 0$,
we say it is unstable.

We shall do some Matlab demo
In next class.