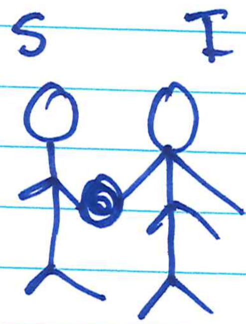
Note

Recovered people cannot get infected again.



Assumptions

(i) Infection is transmitted with probability p.

Pr(~~transmision~~ transmission) = p

(ii) Suppose an individual makes c contacts in a day.

rate of new infection $= I c p \frac{S}{N} = \frac{cp\, IS}{N}$

c – contact rate

p – probability of transmission

S/N – prob. that the contact made is with a susceptible person.

Let $\beta = CP/N$

Then

$$\text{rate of new infection} = \beta I S$$

(iii) Let α be the rate of recovery per day

$\Rightarrow$ $\text{rate of recovery} = \alpha I$

& what is $\frac{1}{\alpha}$?

$\frac{1}{\alpha}$ = time required to recover.

Now, we write our differential equation model

$$\left.\begin{aligned} \frac{dS}{dt} &= -\beta I S \\ \frac{dI}{dt} &= \beta I S - \alpha I \\ \frac{dR}{dt} &= \alpha I \end{aligned}\right\} \text{—— } (*)$$

Total population, $N(t) = S(t) + I(t) + R(t)$

$$\frac{dN(t)}{dt} = 0 \quad \text{(fixed and closed population)}$$

$$\Rightarrow \quad \frac{dS(t)}{dt} + \frac{dI(t)}{dt} + \frac{dR(t)}{dt} = 0$$

Check

$$-\beta IS + (\beta IS - \alpha I) + \alpha I = 0$$

✓

* we can ignore the equation for $R(t)$

since $R(t) = N - (S + I)$

* To solve (*) we need initial conditions, i.e

$$S(0) = N - 1$$

$$I(0) = 1$$

$$R(0) = 0$$

parameters:

Let $N = 10^6$ people

$c = 5$ contacts per day

$p = 0.4$ per contact

$\alpha = 0.3$ per day

Two ~~may~~ main ways of solving diff. Equ.

* analytically
* numerically

why do we solve D.E.s numerically?

* we may have difficult problems that can't be solved by ~~an~~ analytically.
* we want quick solutions
* It enable us to plot the solutions ~~wh~~ which gives insight into the behaviour of the system.

(Lec 0.2, 0.3)

Different Equations (D.E)

D.E is an equation that relates ~~the~~ ~~derivatives~~ a function to its derivatives

Example:

(i) $\frac{df(x)}{dx} = f(x)$

Eg $f(x) = e^x$

$f(x) = \gamma e^x$, γ any constant

$\Rightarrow$ $f(x)$ is a solution of the D.E

(ii) $f'' + 2f' = 3f$

check that e^{-3x} is a solution of the D.E.

Ordinary Differential equations (ODEs):

Contains functions with only one independent variable.

Eg: if we have $f(x)$ or $f(t)$

then $f'' + 2f' = f$

is an ODE.

partial Differential equations (PDEs):

Contains functions with more than one independent variables.

Eg if $f \equiv f(x, y)$, then

$$\frac{\partial f}{\partial x} = \frac{\partial^2 f}{\partial y^2}$$ is a PDE.

Diffusion / Heat equation

ORDER of an ODE is the highest number of derivatives in the equation.

Examples:

(i) $y'' - 4y' + 4y = 0$ — 2nd order

(ii) $y' + \sqrt{y}\, y'''' = \sin(t)$ — 4th order

Linear / Nonlinear ODEs

If an ODE contains only linear functions of $y, y', y'', \ldots$ then it is linear

More precisely,

$a(t) \, dy/dt + b(t)\, y = c(t)$ — 1^{st} order linear

$$a(t) \frac{d^2y}{dt^2} + b(t) \frac{dy}{dt} + c(t)\, y = d(t)$$

— 2nd order linear