Math 215/255 P

Midterm 1, October 18, 2017

Name: **SID:**

Instructor: **Section:**

Instructions

- The total time allowed is 50 minutes.
- The total score is 40 points.
- Use the reverse side of each page if you need extra space.
- Show all your work. A correct answer without intermediate steps will receive no credit.
- Calculators, phones and cheat sheets are not allowed.

Problem	Points	Score
1	5	
2	10	
3	7	
4	6	
5	12	
TOTAL	40	

1. (5 points) Solve the following initial value problems for $y(t)$:

$ty' + 2y = \cos(t), \;\; y\,(\pi) = 0.$

Solution:

2. (10 points) Consider the following system of first order ODEs

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = 2y_1(t) - 2y_2(t)$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = 2y_1(t) + 2y_2(t)$$

(a) Find the general solution of the system. Convert any complex exponentials in your solutions into " real forms" involving sines and cosines.

Solution:

(b) Use the initial conditions $y_1(0) = 1$ and $y_2(0) = 2$ to find the constants in your solution.

Solution:

3. (7 points) Determine the value of k for which the following equation is exact

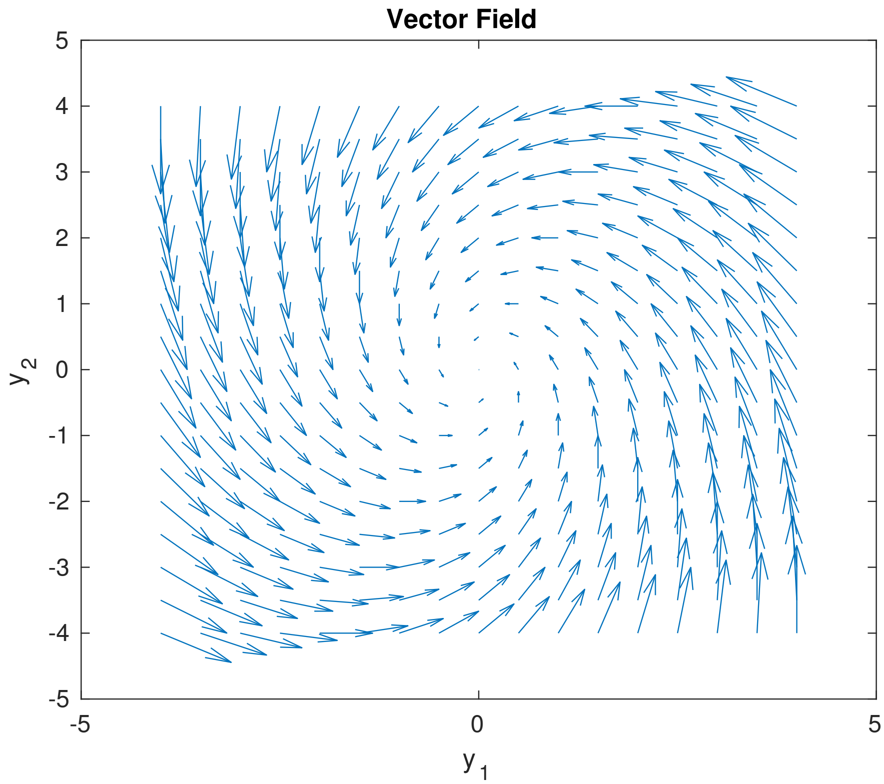
$$(y\cos(x) + kxe^y)\,\mathrm{dx} + (\sin(x) + x^2e^y - 1)\,\mathrm{dy} = 0.$$

Solution:

Use this value of k together with the inital condition $y(\pi) = 0$ to solve the equation.

Solution:

4. (6 points) Suppose the system of equations $\overrightarrow{Y}'(t) = A\overrightarrow{Y}(t)$ has the vector field



(i) This vector field suggests that the eigenvalues associated with the system are;

(a) Distinct real
(b) Repeated real
(c) Complex
(d) None of the above

(ii) Given the initial condition $y_1(0) = 3$ and $y_2(0) = 2$, use the vector field to determine the value of $\overrightarrow{Y}(t)$ as $t \longrightarrow \infty$?

Solution:

(iii) Which of the following Matlab commands will produce this direction field ?

(a)

```
[Y1,Y2] = meshgrid(-4:0.5:4,-4:0.5:4); % creates a meshgrid
U1=  1*Y1 + 1*Y2; %the first equation in the system
U2 = 1*Y1 + 1*Y2; %the second equation in the system
quiver(Y1,Y2,U1,U2) %create the vector field
figure()
quiver(Y1,Y2,U1,U2,2) %create the vector field with longer arrows
title('Vector Field')
xlabel('y_1'), ylabel('y_2')
```

(b)

```
[Y1,Y2] = meshgrid(-4:0.5:4,-4:0.5:4); % creates a meshgrid
U1=  -1*Y1 - 1*Y2; %the first equation in the system
U2 = 2*Y1 - 1*Y2; %the second equation in the system
quiver(Y1,Y2,U1,U2) %create the vector field
figure()
quiver(Y1,Y2,U1,U2,2) %create the vector field with longer arrows
title('Vector Field')
xlabel('y_1'), ylabel('y_2')
```

(c)

```
[Y1,Y2] = meshgrid(-4:0.5:4,-4:0.5:4); % creates a meshgrid
U1=  -2*Y1 + 1*Y2; %the first equation in the system
U2 = 1*Y1 - 2*Y2; %the second equation in the system
quiver(Y1,Y2,U1,U2) %create the vector field
figure()
quiver(Y1,Y2,U1,U2,2) %create the vector field with longer arrows
title('Vector Field')
xlabel('y_1'), ylabel('y_2')
```

(d)

```
[Y1,Y2] = meshgrid(-4:0.5:4,-4:0.5:4); % creates a meshgrid
U1=  1*Y1 + 2*Y2; %the first equation in the system
U2 = 3*Y1 + 2*Y2; %the second equation in the system
quiver(Y1,Y2,U1,U2) %create the vector field
figure()
quiver(Y1,Y2,U1,U2,2) %create the vector field with longer arrows
title('Vector Field')
xlabel('y_1'), ylabel('y_2')
```

5. (12 points) Consider the following ODE

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \lambda(y^2 - 4), \quad \text{where } \lambda > 0\,.$$

(a) Find all the equilibria (steady state solutions) of the differential equation.

Solution:

(b) Sketch the graph of $\frac{\mathrm{d}y}{\mathrm{d}t}$ vs $y(t)$ and use it to determine which of these equilibria is stable, unstable, or semi-stable.

Solution:

(c)Use the initial condition $y(0) = 1$ to find a solution to the equation.
Hint: You may need the partial fraction $\frac{1}{(y-a)(y-b)} = \frac{A}{(y-a)} + \frac{B}{(y-b)}$.

Solution:

(d) Find the limit of this solution $y(t)$ as $t \longrightarrow \infty$?

Solution: