Math 215/255
Midterm 2, Nov 15, 2017

Name: **SID:**

Instructor: **Section:**

Instructions

- The total time allowed is 50 minutes.
- The total score is 40 points.
- Use the reverse side of each page if you need extra space.
- Show all your work. A correct answer without intermediate steps will receive no credit.
- Calculators, phones and cheat sheets are not allowed.

Problem	**Points**	**Score**
1	14	
2	12	
3	14	
TOTAL	40	

1. (14 points)

a) (5 points) Find a general form of homogeneous solutions for the equation

$$\frac{d}{dt}\vec{x}(t) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \vec{x}(t).$$

Solution:

b) (2 points) For which initial conditions will the solution remain bounded for large t.

Solution:

c) (7 points) Find a particular solution to

$$\frac{d}{dt}\vec{x}(t) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 3t\mathrm{e}^{-t} \\ -3t\mathrm{e}^{-t} \end{bmatrix}.$$

Solution:

2. (12 points) A damped spring-mass system has mass 4 kg, friction constant 2 kg/s, and spring constant k kg/s^2.

   a) (4 points) For what values of $k \geq 0$ is the spring under-damped, over-damped, and critically-damped?

   Solution:

   b) (5 points) For $k = 2$ a force is applied of $3\sin(t)$ kg m/s^2, compute a particular solution of the damped spring-mass equation.

   Solution:

   c) (3 points) If there was no friction, but still mass 4 kg and spring constant $k = 2$, give a forcing term that would exhibit resonance.

   Solution:

3. (14 points) The following questions concern the equation

$$\frac{d}{dt}\vec{y}(t) = \begin{bmatrix} 0 & 1 \\ -12 & -8 \end{bmatrix} \vec{y}(t) + \begin{bmatrix} 0 \\ f(t) \end{bmatrix}.$$

a) (4 points) Find a fundamental matrix for the homogeneous part of the equation.

Solution:

b) (6 points) Given the particular solution $\vec{y}_P(t) = \begin{bmatrix} te^{-2t} \\ e^{-2t} - 2te^{-2t} \end{bmatrix}$, find the forcing term $f(t)$.

Solution:

c) (4 points) Solve for $\vec{y}(t)$ with the forcing of part b) and the initial conditions

$$\vec{y}(0) = \begin{bmatrix} 0 \\ -3 \end{bmatrix}.$$

Solution: