In general, given an autonomous ODE of the form

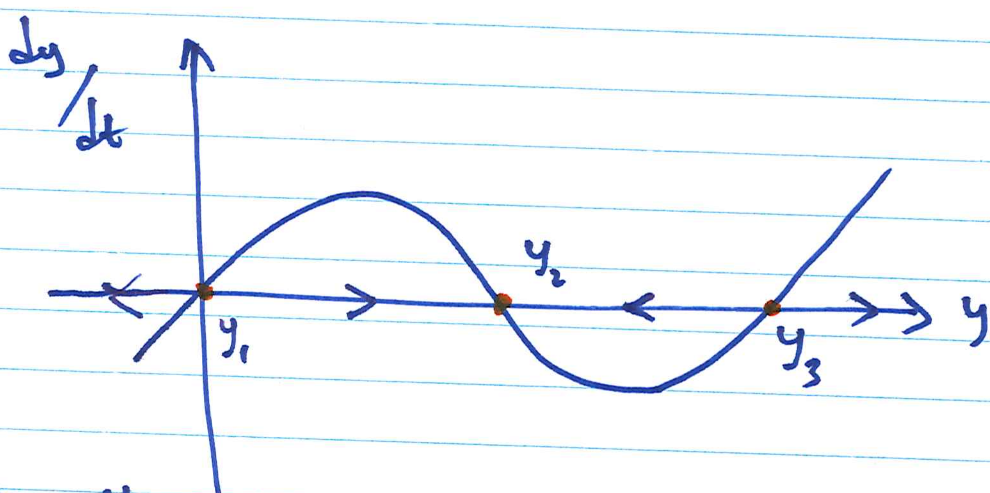
$$dy/dt = f(y)$$

* To get the steady-state solutions, set

$$dy/dt = 0$$

and solve $f(y_{ss}) = 0$ for y_{ss}.

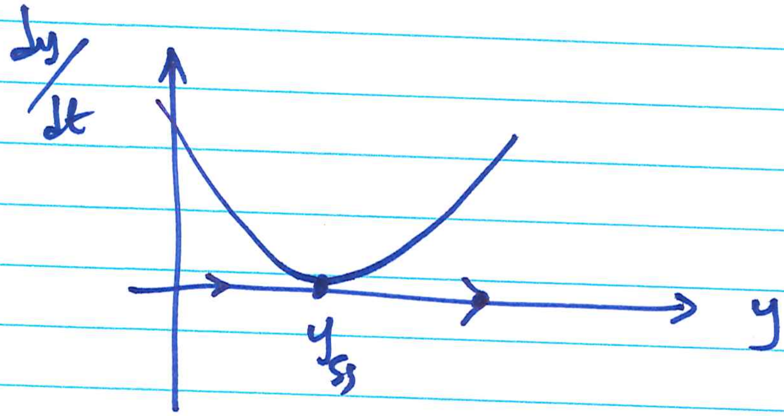
* plot of dy/dt vs y



$\Rightarrow$ y_2 is <u>stable</u>

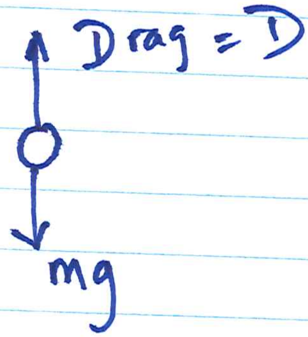
$\Rightarrow$ y_1 and y_3 are <u>unstable</u>

suppose we have



$\Rightarrow$ $y = y_{ss}$ is semi-stable.

Example: Small falling object



From Newton's second law of motion,

Sum of forces = ~~max~~ ~~a.~~ mass x acceleration

$$mg - D = ma$$

But $dv/dt = a$,

$$mg - D = m\,dv/dt$$

Also for small objects

$$D \simeq kv$$

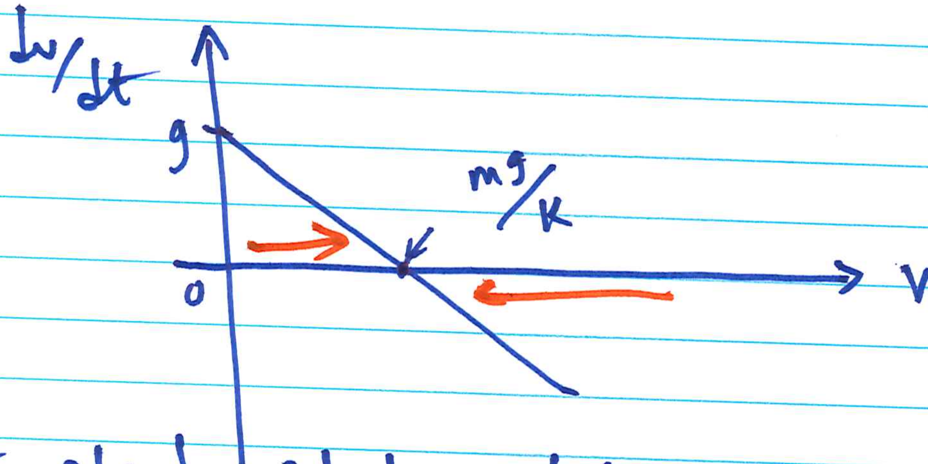
where k depends ~~of~~ on geometry, — .

$$\Rightarrow \quad mg - kv = m\,dv/dt$$

We have a 1st order linear ODE.

$$dv/dt = g - \frac{k}{m} v$$

Let us plot dv/dt vs v



for steady-state solution, set

$$dv/dt = 0$$

$$\Rightarrow \quad g - \frac{k}{m} v = 0$$

$$\Rightarrow \quad v = \frac{mg}{k}, \quad \text{"terminal velocity"}$$

Now let us solve the ODE first,

$$dv/dt = g - \frac{k}{m} v$$

$$\int \frac{dv}{\left(g - \frac{k}{m} v\right)} = \int dt + c_1$$

$$-\frac{1}{k/m} \ln \left| g - \frac{k}{m} v \right| = t$$

Let us solve the integral

$$\int \frac{dv}{(g - k/m\, v)}$$

Let $u = g - k/m\, V$

$du = -k/m\, dv$

$dv = -m/k\, du$

$$\int \frac{1}{u}\left(-m/k\right) du = -m/k \int \frac{1}{u}\, du = -m/k \ln|u|.$$

$$\Rightarrow \quad -m/k \ln\left|g - k/m\, v\right| = t + C_1$$

$$\ln\left|g - k/m\, v\right| = -\frac{k}{m}t - \frac{k}{m}C_1$$

$$\ln\left|g - k/m\, v\right| = -\frac{k}{m}t + C_2 \qquad \left(C_2 = \frac{-k}{m}C_1\right)$$

$$g - k/m\, v = C_3 e^{-k/m\, t} \qquad \left(C_3 = e^{C_2}\right)$$

$$V = \frac{m}{k}\left(g - c_3 e^{-k/m\, t}\right)$$

$$V = \frac{mg}{k}\left(1 - \frac{c_3}{g} e^{-k/m\, t}\right)$$

$$V = \frac{mg}{k}\left(1 - c_4 e^{-k/m\, t}\right)$$