Math215/255 Section 104 Quiz 1 (15 Minutes)

Name: Solution Student Number:

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Question 1:

Solve the IVP

$$y' = \frac{7}{5}(y-3)^{2/7}, \qquad y(1) = 3$$

Is the solution unique near $(t, y) \equiv (1, 3)$? Justify your answer.

$$dy/dt = 7/5\,(y-3)^{2/7} \quad \text{———} \quad (1)$$

$$\int \frac{dy}{(y-3)^{2/7}} = \int 7/5\, dt + C_1$$

$$\int (y-3)^{-2/7}\, dy = \frac{7}{5}t + C_1$$

$$\frac{(y-3)^{5/7}}{5/7} = 7/5\, t + C_1$$

multiplying through by 5/7, we have

$$\Rightarrow (y-3)^{5/7} = t + C_2 \qquad (C_2 = 5/7\, C_1)$$

$$y(1) = 3 \Rightarrow 0 = 1 + C_2 \Rightarrow C_2 = -1$$

$$(y-3)^{5/7} = t - 1$$

Observe that $y(t) = 3$ is also a solution.

Substitute $y = 3$ into (1), we also have $dy/dt = 0$

$dy/dt = 0$,

$y(1) = 3$ $\therefore y(t) = 3$ Satisfy the IVP.

$\boxed{\text{Solution not unique near } (1,3)!}$

Question 2:

Solve the IVP

$$\frac{dy}{dt} + \frac{2y}{t} = \frac{\sin(t)}{t^2}, \qquad y(\pi) = \frac{2}{\pi^2} \quad (1)$$

Show the details of your solution.

$p(t) = 2/t \quad , \quad \varphi(t) = \frac{\sin(t)}{t^2}$

$h(t) = e^{\int p(t)\,dt} = e^{\int \frac{2}{t}\,dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

multiply through (1) by t^2.

$t^2 \, dy/dt + 2ty = \sin(t)$

$\frac{d}{dt}\left(t^2 y(t)\right) = \sin(t)$

$\int d\left(t^2 y(t)\right) = \int \sin(t)\,dt + C_1$

$t^2 y(t) = -\cos(t) + C_1$

$y(t) = -\frac{\cos(t)}{t^2} + \frac{C_1}{t^2}$

$y(\pi) = \frac{2}{\pi^2}$ $\quad \boxed{\cos(\pi) = -1}$

$\Rightarrow \frac{2}{\cancel{\pi^2}} = -\frac{\cos(\pi)}{\cancel{\pi^2}} + \frac{C_1}{\cancel{\pi^2}} \Rightarrow C_1 = 2 - 1$

$C_1 = 1$

$\therefore y(t) = \frac{1}{t^2}\left(1 - \cos(t)\right)$