Math215/255 Section 104 Quiz 2 (15 Minutes)

Name: Solution Student Number:

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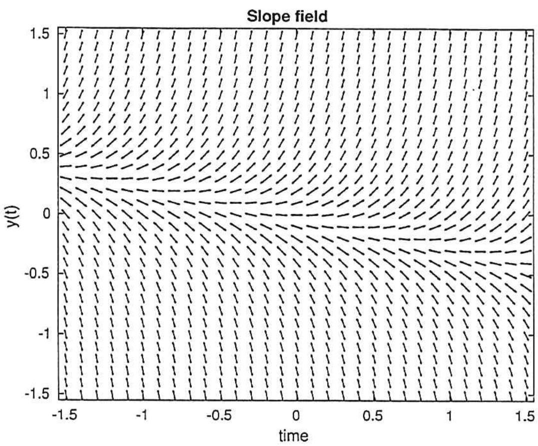
Instructions: Answer ALL three questions.

Question One:

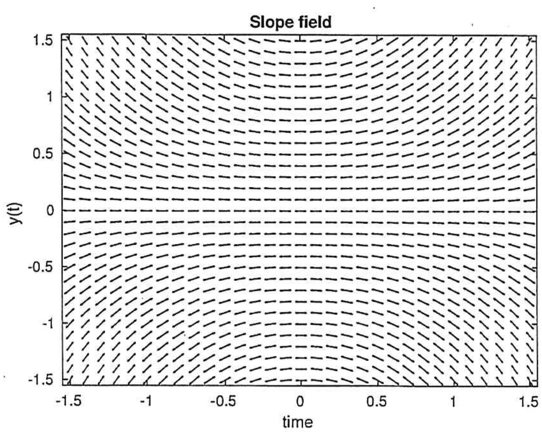
(1) This slope field indicates that the associated differential equation has which form



C

(a) $y(t)' = f(t)$
(b) $y(t)' = f(y)$
(c) $y(t)' = f(t, y)$
(d) None of the above

(2) Which of the following differential equation matches the slope field



C

(a) $y(t)' = t - y$
(b) $y(t)' = y/t$
(c) $y(t)' = ty$
(d) $y(t)' = y$

Question Two:

Consider the initial value problem (IVP)

$$\frac{dy}{dt} = 3 - 2t - y, \qquad y(0) = 1,$$

(a) To approxiate this differential equation using Euler's method, what difference formula would you get?
(b) With $h = 0.1$, use the difference formula to approximate $y(0.2)$.
Show the details of your solution.

(a) Euler method is given by

$$y_{n+1} = y_n + h f(t_n, y_n)$$

But $f(t_n, y_n) = 3 - 2t_n - y_n$

$$y_{n+1} = y_n + h(3 - 2t_n - y_n)$$

$$\boxed{y_{n+1} = (1-h)y_n + (3 - 2t_n)h}$$ ← difference formula.

(b) when $n = 0$,

$$y_1 = (1-h)y_0 + (3 - 2t_0)h$$

$$= (1 - 0.1)1 + (3 - 2(0))0.1$$

$$y_1 = 0.9 + 0.3 = 1.2$$

$$y_2 = (1-h)y_1 + (3 - 2t_1)h$$

$$= (1 - 0.1)1.2 + (3 - 2(0.1))0.1$$

$$y_2 = 1.36$$

$$\boxed{y(0.2) \simeq 1.36}$$

Question Three:

Consider the following first order ODE

$$(3xy + y^2)\ dx + (x^2 + xy)\ dy = 0 \quad \text{1}$$

(a) Is the equation exact?
(b) If yes, find the general solution. Otherwise, find an integrating factor $h(x)$ and use it to find the general solution of the equation.
(c) Find the constant in the general solution using $y(1) = 1$.
Show the details of your solution.

$M(x,y) = 3xy + y^2, \quad N(x,y) = x^2 + xy$

$M_y = 3x + 2y \neq 2x + y = N_x$

$\therefore M_y \neq N_x$

$\therefore$ The equation is not exact.

To find $h(x)$, we need to solve

$$\frac{dh(x)}{dx} = \left(\frac{M_y - N_x}{N}\right) h(x) = \left[\frac{(3x+2y) - (2x+y)}{x^2+xy}\right] h(x)$$

$$= \left[\frac{\cancel{(x+y)}}{x\cancel{(x+y)}}\right] h(x) = \left(\frac{1}{x}\right) h(x)$$

$$\frac{dh(x)}{dx} = \left(\frac{1}{x}\right) h(x)$$

$$\frac{dh(x)}{h(x)} = \left(\frac{1}{x}\right)$$

integrating, $\ln h(x) = \ln(x) + C_1$

$h(x) = C_2 e^{\ln(x)} = C_2 x$, $\quad$ (Set $C_2 = 1$)

$h(x) = x$

Multiply ~~equation~~ $h(x) = x$ through equation (1)

$\Rightarrow (3x^2y + xy^2)\,dx + (x^3 + x^2y)\,dy = 0$

$M(x,y) = 3x^2y + xy^2 \quad , \quad N(x,y) = x^3 + x^2y$

Let $\psi(x,y) = C$

$\frac{\partial\psi}{\partial x} = 3x^2y + xy^2$

Integrating, $\psi(x,y) = x^3y + \frac{1}{2}x^2y^2 + \gamma_1(y)$ —— (2)

$\frac{\partial\psi}{\partial y} = x^3 + x^2y$

integrating, $\psi(x,y) = x^3y + \frac{1}{2}x^2y^2 + \gamma_2(x)$ —— (3)

equating (2) and (3), we have

$\gamma_1(y) = \gamma_2(x) = 0$

$\therefore \boxed{\psi(x,y) = x^3y + \frac{1}{2}x^2y^2}$ —— general solution

$\therefore$ the general solution is $x^3y + \frac{1}{2}x^2y^2 = C$

$y(1) = 1 \quad \Rightarrow 1 + \frac{1}{2} = C$

$\Rightarrow C = 3/2$

$$\boxed{x^3y + \frac{1}{2}x^2y^2 = 3/2}$$